BeFo

STIFTELSEN BERGTEKNISK FORSKNING
ROCK ENGINEERING RESEARCH FOUNDATION

RELIABILITY-BASED DESIGN OF ROCK TUNNEL SUPPORT

William Bjureland

BeFo Report 209

STIFTELSEN BERGTEKNISK FORSKNING
ROCK ENGINEERING RESEARCH FOUNDATION

RELIABILITY-BASED DESIGN OF ROCK TUNNEL SUPPORT

Tillförlitlighetsbaserad dimensionering av bergtunnelförstärkning

William Bjureland

BeFo Report 209
Stockholm 2024
ISSN 1104-1773
ISRN BEFO-R-209-SE

This report is a representation of the Doctoral Thesis, KTH Royal Institute of Technology School of Architecture and the Built Environment Department of Civil and Architectural Engineering Division of Soil and Rock Mechanics © William Bjureland TRITA-ABE-DLT-208 ISBN 978-91-7873-522-8. This BeFo report is published with the permission of the author.

PREFACE

Design of rock tunnels can be performed in accordance with the Eurocode, which allows that different design methodologies are applied, such as design by calculation or design using the observational method. To account for uncertainties in design, the Eurocode states that design by calculation should primarily be performed using the partial factor method or reliability-based methods. The basic principle of both of these methods is that it shall be assured that a structure's resisting capacity is larger than the load acting on the structure, with sufficiently high probability. Even if this might seem straightforward, the practical application of limit state design to rock tunnel support has only been studied to a limited extent.

The research presented in this report focuses on the above and was performed between the end of 2014 and the beginning of 2020 at the Division of Soil and Rock Mechanics, Department of Civil and Architectural Engineering, at KTH Royal Institute of Technology in Stockholm, Sweden and is in much a copy of the doctoral thesis by the author.

The doctoral work was supervised by Fredrik Johansson, Stefan Larsson, and Johan Spross. Many of the underlying reports was co-written with Anders Prästings, Andreas Sjölander and. Håkan Stille. The input from the reference group that included Tommy Ellison, Mats Holmberg, Diego Mas Ivars, Cecilia Montelius, Jonny Sjöberg, Håkan Stille, Robert Sturk, Per Tengborg, and Lars-Olof Dahlström, is gratefully acknowledged.

The work herein presented was funded by BeFo, SBUF, SVC, SKB and KTH.

Stockholm

Patrik Vidstrand

FÖRORD

Design av bergtunnlar kan utföras i enlighet med Eurokoden, som tillåter att olika designmetoder tillämpas, såsom design genom beräkning eller design med observationsmetoden. För att ta hänsyn till osäkerheter i designen anger Eurokoden att design genom beräkning i första hand ska utföras med partialfaktormetoden eller med tillförlitlighetsbaserade metoder. Grundprincipen för båda dessa metoder är att det ska säkerställas att en konstruktions lastbärande förmåga är större än den belastning som verkar på konstruktionen; detta med tillräckligt stor sannolikhet. Även om detta kan tyckas okomplicerat, har den praktiska tillämpningen av gränstillståndsdesign på bergtunnelstöd endast studerats i begränsad omfattning.

Forskningen som presenteras i denna rapport fokuserar på ovanstående och utfördes mellan 2014 och 2020 vid Kungliga Tekniska Högskolan i Stockholm och rapporten här är i stort sett en kopia av doktorsavhandlingen av författaren.

Doktorandarbetet handleddes av Fredrik Johansson, Stefan Larsson och Johan Spross. Många av de bakomliggande rapporterna skrevs tillsammans med Anders Prästings, Andreas Sjölander och. Håkan Stille. Stödet från referensgruppen som inkluderade Tommy Ellison, Mats Holmberg, Diego Mas Ivars, Cecilia Montelius, Jonny Sjöberg, Håkan Stille, Robert Sturk, Per Tengborg och Lars-Olof Dahlström var värdefullt.

Arbetet som presenteras här har finansierats av BeFo, SBUF, SVC, SKB och KTH.

Stockholm

Patrik Vidstrand

SUMMARY

Since 2009, design of rock tunnels can be performed in accordance with the Eurocode, which allows that different design methodologies are applied, such as design by calculation or design using the observational method. To account for uncertainties in design, the Eurocode states that design by calculation should primarily be performed using the partial factor method or reliability-based methods. The basic principle of both of these methods is that it shall be assured that a structure's resisting capacity is larger than the load acting on the structure, with sufficiently high probability. Even if this might seem straightforward, the practical application of limit state design to rock tunnel support has only been studied to a limited extent.

The overall aim of this project has been to develop reliability-based methods for environmental and economic optimization of rock tunnel support, with a special focus on shotcrete support. To achieve this, this report aims to: (1) assess the applicability of the partial factor method and reliability-based methods for design of shotcrete support, exclusively or in combination with the observational method, (2) quantify the magnitude and uncertainty of the shotcrete's input parameters, and (3) assess the influence from spatial variability on shotcrete's load-bearing capacity and judge the correctness of the assumption that the load-bearing capacity of the support is governed by the mean values of its input parameters.

The results shows that the partial factor method is not suitable, and in some cases not applicable, to use in design of rock tunnel support. Instead, a reliability-based design methodology for shotcrete in rock tunnels with respect to loose blocks between rockbolts and a design methodology for shotcrete lining based on a combination of the observational method is suggested. The presented design methodologies enable optimization of the shotcrete support and shotcrete lining by stringently accounting for uncertainties related to input data throughout the design process. The report also discusses the limited knowledge that we as an industry sometimes have in our calculation models and the clarifications that should be made in future revisions of the Eurocode related to target reliability and the definition of failure.

Keywords: Rock engineering, reliability-based design, Eurocode 7, observational method, tunnel engineering

SAMMANFATTNING

Sedan 2009 kan dimensionering av bergtunnlar utföras i enlighet med Eurokoden, vilken tillåter att olika dimensioneringsmetoder tillämpas, så som dimensionering genom beräkning eller dimensionering med observationsmetoden. För att ta hänsyn till osäkerheter föreskriver Eurokoderna att dimensionering genom beräkning primärt skall utföras med hjälp av partialkoefficientmetoden eller tillförlitlighetsbaserade metoder. Grundprincipen i båda dessa metoder är att det skall säkerställas att en konstruktions bärförmåga med tillräckligt hög sannolikhet, är större än lasten som verkar mot konstruktionen. Även om detta kan förefalla enkelt så har den praktiska användningen av framförallt tillförlitlighetsbaserade metoder inom bergbyggande endast studerats i begränsad utsträckning.

Målet med detta projekt har varit att utveckla tillförlitlighetsbaserade metoder för miljömässig och ekonomisk optimering av förstärkning i tunnlar med fokus på sprutbetongförstärkning. För att uppnå detta, syftar denna rapport till att (1) utvärdera tillämpbarheten av partialkoefficient metoden och tillförlitlighetsbaserade metoder för dimensionering av sprutbetongförstärkning, (2) kvantifiera storleken och osäkerheten i sprutbetongförstärkningens indata parametrar och (3) utvärdera effekten från rumslig spridning på sprutbetongens bärförmåga. Resultaten visar att partialkoefficientmetoden inte är lämplig att använda vid dimensionering av förstärkning i tunnlar. En tillförlitlighetsbaserad dimensioneringsmetodik för sprutbetong med avseende på blockutfall mellan bultar samt en dimensioneringsmetodik för tunnellining av sprutbetong baserad på observationsmetoden och tillförlitlighetsbaserade metoder har utvecklats inom ramen av detta arbete. De utvecklade metodikerna möjliggör optimering av förstärkning och tunnellining av sprutbetong genom att stringent ta hänsyn till osäkerheter kopplade till indata kontinuerligt genom hela designprocessen. Rapporten diskuterar även den begränsade kunskap vi har om våra beräkningsmodeller samt vilka förtydliganden som bör göras i framtida revideringar av Eurokoden kopplade till riktvärden förkravställda brottsannolikheter och definitionen av brott.

Nyckelord: Bergmekanik, sannolikhetsbaserad dimensionering, Eurokod 7, observationsmetoden, tunnelbyggnad.

CONTENTS

1. INTRODUCTION

1.1 Background

In both cities and rural areas, tunnels and caverns are excavated for a number of purposes, such as metro lines, roads, railways, sewage systems, hydropower plants, mines, and nuclear waste deposits. Regardless of the intended application, underground excavation in rock involves great uncertainties that must be stringently accounted for during design and construction to ensure that society's requirements of structural safety is fulfilled while its environmental and economic impact is minimized.

Design of underground excavations in rock (hereinafter referred to as rock tunnels) can be performed with a number of rock engineering design tools, such as classification systems, numerical or analytical calculations, the observational method, and engineering judgement (Palmstrom & Stille 2007). Depending on the failure mode expected and the incorporated uncertainties, different tools are suitable to use in the design.

Historically, design using calculations and the deterministic total safety factor approach have played an important role in design codes for management of uncertainties and verification of structural safety. Since 2009, however, verification of structural safety in civil engineering shall, according to the European commission, in countries within the European Union, EU, be performed in accordance with the European design standards, the Eurocodes (CEN 2002). The Eurocodes are a collection of design standards applicable to most structures and materials of civil engineering: some examples of standards related to this thesis are basis of design (EN1990), concrete (EN1992), steel (EN1993), and soil and rock (EN1997).

The basic rule in the Eurocodes is that for all design situations it must be verified that no relevant limit state is attained. For such verifications, in each Eurocode, a number of different accepted limit state verification methods are specified. In EN1990 (CEN 2002) the specified methods are structural analysis and design assisted by testing. In Eurocode 7 (CEN 2004), the specified methods are design by calculation, prescriptive measures, load tests and tests on experimental models, and the observational method (Figure 1.1).

For design of rock tunnel support, limit state verification can in many situations be performed using calculations (Palmstrom & Stille 2007). Eurocode 7 suggests that analytical, semi-empirical, or numerical calculation models are appropriate for such calculations (Figure 1.1). To account for uncertainties, the Eurocodes recommend that limit states are verified using "the partial factor method". The partial factor method is a reliability-based design method that stringently accounts for uncertainties by increasing the calculated load and decreasing the calculated resistance through application of partial factors on their respective characteristic values. The increased load and decreased resistance are usually referred to as design values. Structural safety is ensured by verifying that the design value of the load is smaller or equal to the design value of the resistance; thus, creating a margin of safety against limit state attainment. The Eurocodes' version of the partial factor method, however, incorporate fixed partial factors for specific materials. Thereby, a part of the advantages of the method is possibly lost.

As an alternative to their own version of the partial factor method, the Eurocodes accept the use of reliability-based design methods. In such methods, uncertainties are stringently accounted for by assigning representative probability distributions to all relevant uncertain input parameters. Structural safety is ensured by verifying that the probability of limit state attainment, that is the probability that the load will exceed the resistance, is sufficiently low for every relevant limit state.

However, there is a lack of knowledge regarding the magnitude and uncertainty of the input parameters usually incorporated in design of rock tunnel support. In addition, the spatial variability of the input parameters is commonly ignored by assuming that the load-bearing capacity of the tunnel support is governed by the mean values of its input parameters. Lastly, the uncertainties incorporated in design and construction of rock tunnels are to a large extent epistemic; that is, they are due to a lack of knowledge. Therefore, limit state verification using calculations and reliability-based methods solely might not always be suitable. In such cases, alternative approaches or additional measures to ensure structural safety are necessary.

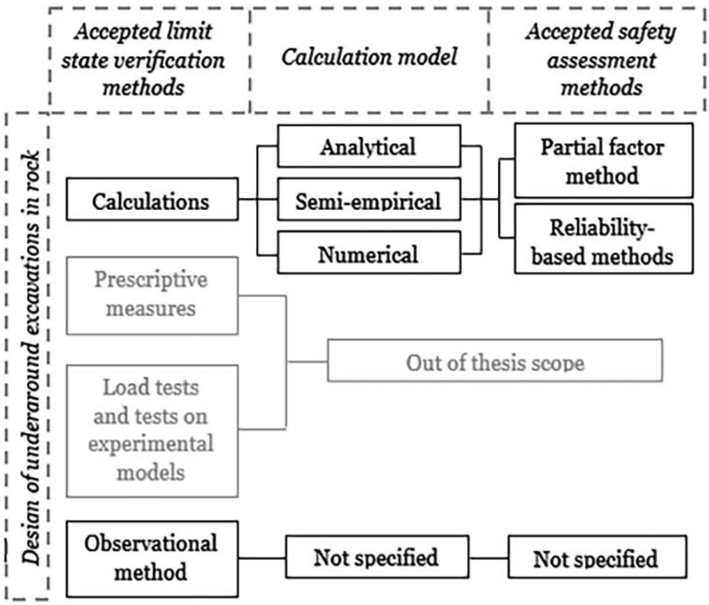

Figure 1.1. Accepted limit state verification tools available to the rock engineer along with suggested calculation models and accepted safety assessment methods.

One such alternative approach is to apply the observational method and incorporate monitoring during construction into the design process. In the observational method, the main idea is to predict the behavior of a structure, before construction is started, and through monitoring during construction assess the structure's behavior. However, in its current form, Eurocode 7 (CEN 2004) gives no recommendations, or limitations, on how the requirements of the observational method stated in Eurocode 7 (CEN 2004) shall be fulfilled in practical design situations. It is clear, however, that incorporation of calculations, which stringently account for uncertainty in parameters, are needed in order to fulfill the formal requirements of the observational method. Therefore, to effectively account for and decrease the incorporated uncertainties, an attractive approach would be to use reliability-based calculations within the framework of the observational method.

The overall aim of this project was to develop reliability-based design methods for environmental and economic optimization of rock tunnel support, with a special focus on shotcrete support. By doing so, optimization of the support, with respect to the incorporated uncertainties, might be possible without compromising on society's requirements of structural safety.

To achieve this, the specific aims of this thesis are to: (1) assess the applicability of the partial factor method and reliability-based methods for design of shotcrete support, exclusively or in combination with the observational method, (2) quantify the magnitude and uncertainty of the shotcrete's input parameters, and (3) assess the influence from spatial variability on shotcrete's load-bearing capacity and judge the correctness of the assumption that the load-bearing capacity of the tunnel support is governed by the mean values of its input parameters; that is, it acts as an averaging system.

The major part of the work has been performed as case studies. For these reasons, the content and conclusions are all related to rock engineering design and are mainly focused on the specific findings of the studied cases.

As previously mentioned, prescriptive measures, load tests, and tests on experimental models, are accepted limit state verification methods according to Eurocode 7 (CEN 2004). However, they are all outside the scope of this thesis.

2. Design of rock tunnel support

2.1 Introduction

For design of rock tunnel support, there are a number of failure modes, or limit states, that need to be considered. These limit states can essentially be divided into two main types: (I) limit states in which the load, S, and the resistance, R, can be separated and (II) limit states in which such a distinction cannot easily be made, because some input parameters are incorporated in both S and R (Johansson et al. 2016). The relevant type of limit state in each design situation depends on aspects such as the type of rock mass, the stress conditions, and the depth and geometry of the excavation.

In the following a brief presentation is made on common rock engineering design applications of shotcrete support related to both type (I) and type (II).

2.2 Limit states with separable load and resistance

As mentioned, the common feature for limit states of type (I) is that, after simplifications, a distinction can be made between the parameters affecting S and the parameters affecting R (Bagheri 2011). Considering for example the limit states, or failure modes, presented in the Swedish Transport Administration's design guidelines (Lindfors et al. 2015), some common design issues of type (I) are suspension of a loose core of rock mass using rock bolts and gravity loaded arch (Johansson et al. 2016).

Another failure mode of type (I), which must commonly be accounted for in design of tunnels in jointed rock, is loose blocks that can fall or slide into the underground opening. The analysis of unstable blocks and the design of support measures to secure them have been studied by numerous authors (e.g. Hoek & Brown 1980, Goodman & Shi 1985, Mauldon 1990, Mauldon & Goodman 1990, Hatzor 1992, Mauldon 1992, Mauldon 1993, Mauldon & Goodman 1996, Tonon 1998, Tonon 2007, Bagheri 2011, Brady & Brown 2013).

A common support measure for loose blocks is to apply a thin shotcrete layer to the periphery of the excavation and to systematically install rockbolts into the surrounding rock mass. The main idea of this support system is that larger blocks are secured by the rockbolts and smaller blocks, which can fit between the rockbolts, are secured by the shotcrete.

The shotcrete's ability to secure these smaller blocks is to a large extent governed by the existence of sufficient adhesion in rock–shotcrete interface along the circumference of the block (Figure 2.1).

To ensure that the structural capacity of the shotcrete is sufficient, analytical calculations are commonly used. Using analytical calculations, the shotcrete's adhesive capacity, R_{a}, to sustain a loose block can be calculated as (Barrett and McCreath, 1995):

$$R_a = a\delta O, \quad (1)$$

where a is the adhesion, δ is the width of the load bearing zone along the circumference, O, of the block (Figure 2.2 a). The R_a is sufficient if it exceeds the potential, W, of the loose block:

$$W = V\gamma_r, \quad (2)$$

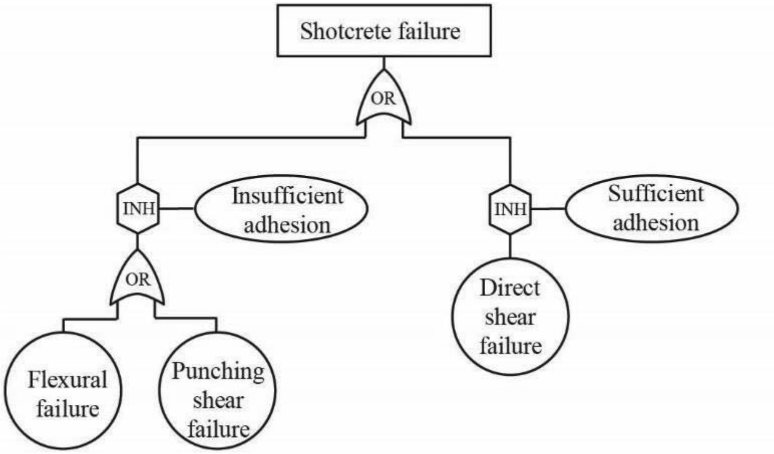

Figure 2.1. Fault tree representing the structural system of shotcrete support (© Bjureland et al. 2019, CC–BY 4.0, https://creativecommons.org/licenses/by/4.0).

in which V is the volume of the block and γ_r is the unit weight of the rock mass. If the R_a is sufficient, the shotcrete's capacity is then governed by its direct shear capacity, $R_{d.sh}$, (Barrett and McCreath, 1995):

$$R_{d.sh} = f_{sh}tO, \quad (3)$$

in which f_{sh} is the direct shear strength of the shotcrete (Figure 2.2 b)). The $R_{d.sh}$ is sufficient if it is larger than W. If the R_a is insufficient, and the shotcrete debonds from the rock surface, the shotcrete must instead support the block through its punching shear resistance, $R_{p.sh}$, and its bending moment capacity, R_{fl} (Figure 2.1). In the former case, failure of the shotcrete occurs at the location of the rockbolts where shear forces are at their maximum and the rockbolts' face plates therefore punches through the shotcrete layer when the shotcrete is exposed to a load (Barrett and McCreath, 1995) (Fig. 2.2c)). Punching failure of the rokbolts's face plates occurs at an inclined plane along the circumference of the face plate. However, following the common practice of assuming that failure occurs along an equivalent vertical plane situated at a distance of $(2b + t)/2$

from the rockbolts, where b is the equivalent radius of the face plate (Holmgren, 1992; Barrett and McCreath, 1995), the $R_{\mathrm{p.sh}}$ can be calculated as (Holmgren, 1992):

$$R_{\mathrm{p.sh}} = f_{\mathrm{sh}} \pi t(2b + t). \tag{4}$$

Similar to $R_{\mathrm{d.sh}}$, the $R_{\mathrm{p.sh}}$ is sufficient if it exceeds W.

The R_{fl} can be calculated using different approaches, depending on whether plain or fibre-reinforced shotcrete is used. If plain shotcrete is used, one approach is to estimate the R_{fl} based on its capacity at first crack, that is when its elastic limit is reached and thus when its bending tensile capacity, $f_{\mathrm{ctm,fl}}$, is exceeded (Banton et al., 2004). The R_{fl} per meter width of the shotcrete layer can then be calculated as (e.g. Barrett and McCreath, 1995; Banton et al., 2004):

$$R_{\mathrm{fl}} = \frac{f_{\mathrm{ctm,fl}} t^2}{6}. \tag{5}$$

If fibre-reinforced shotcrete is used, a common approach is to estimate the R_{fl} by accounting for the increased toughness introduced by the fibres as (Holmgren, 1992):

$$R_{\mathrm{fl}} = 0.9 \frac{R_{10/5} + R_{30/10}}{200} \frac{f_{\mathrm{ctm,fl}} t^2}{6}, \tag{6}$$

in which $R_{10/5}$ and $R_{30/10}$ are flexural toughness factors (ASTM, 1997). In principle, these flexural toughness factors adjust the moment capacity of the shotcrete material to account for the residual strength provided by the fibers. Thereby, they provide information regarding the shotcrete's performance compared to an elastic perfectly plastic shotcrete (Holmgren, 1992). For an elastic perfectly plastic material, both $R_{10/5}$ and $R_{30/10}$ are equal to 100. The factor 0.9 is introduced to account for the overestimation of R_{fl} that Eq. 6 otherwise yields at small deflections for a shotcrete with a relatively high residual strength (Holmgren, 1992). The R_{fl} is sufficient if it is larger than the potential bending moment, M, in the shotcrete caused by the load from the loose block (Fig. 2.2d).

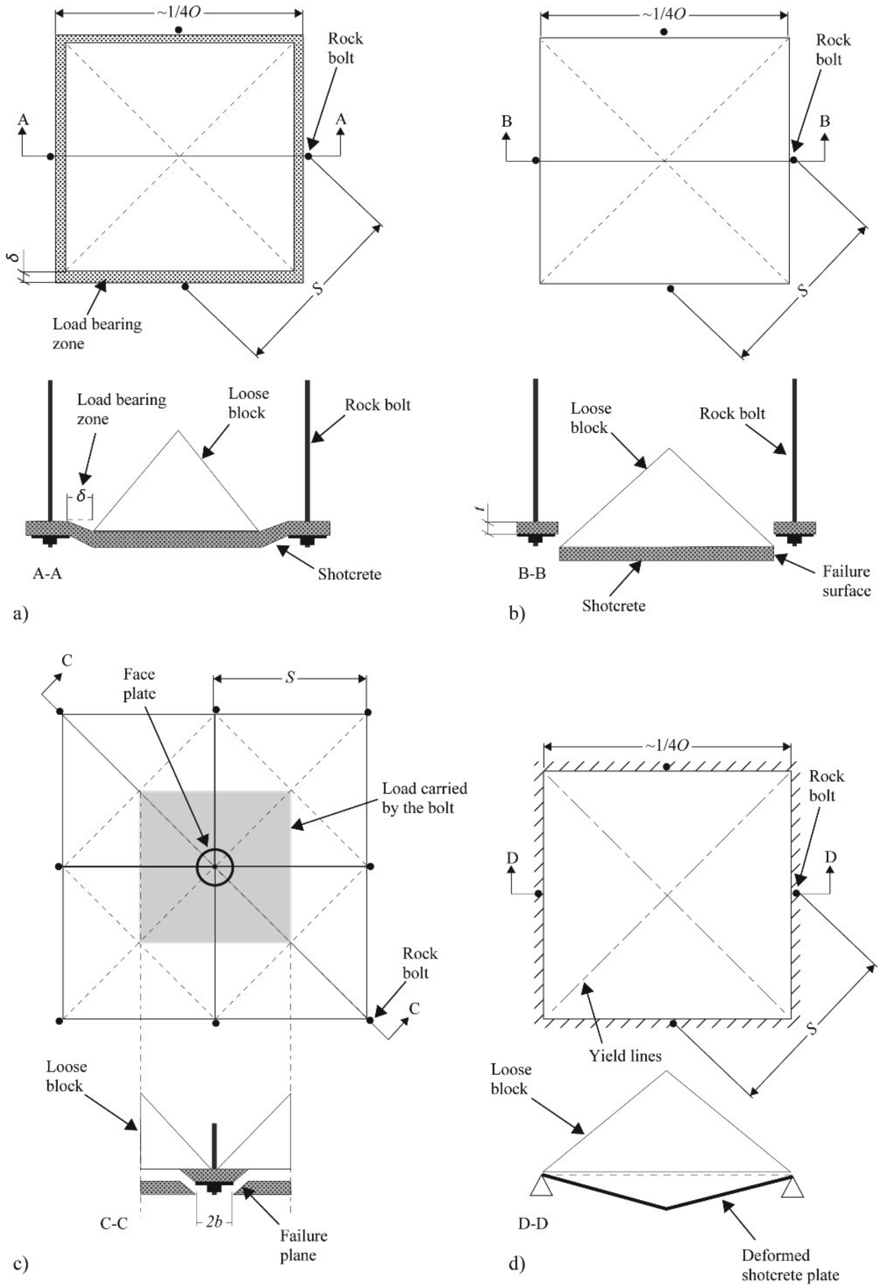

Figure 2.2. a) Adhesive failure model; b) Direct shear failure model; c) Punching shear failure model; d) Flexural failure model. (© Bjureland et al. 2019, CC–BY 4.0, https://creativecommons.org/licenses/by/4.0).

2.3 Limit states with interaction between load and resistance

For limit states of type (II), a clear distinction between the load and the resistance cannot easily be made. As an example, the convergence–confinement method (e.g. Brown et al. 1983), is a typical case in which it might be difficult to derive how different uncertain parameters affect the behavior of the analyzed structure.

The convergence–confinement method is a graphical solution that describes the development of radial peripheral deformations in a deeply situated circular tunnel with a radius, r, during excavation (Figure 2.3). The deformations develop as a result of the change in stress state in the surrounding rock mass. Assuming an elastic–plastic rock mass with a Mohr–Coulomb failure criterion and a non-associated flow rule for the dilatancy after failure (Stille et al. 1989), illustratively, consider a cross-section along the progression line of a deeply situated circular tunnel. Before excavation is started, a certain initial stress state, p_0, supporting the imaginary periphery of the planned tunnel is present in the rock mass. When excavation has been initiated and the face of the excavation approaches the considered cross-section, the supportive initial stresses starts to decrease. For small changes in the stress state, i.e. at some distance before the excavation reaches the cross section, elastic radial deformations of the tunnel surface, u_{ie}, develops due to the decrease in supportive radial pressure, p_{i}, acting on the tunnel periphery. The magnitude of the u_{ie} can be calculated as:

$$u_{\mathrm{ie}} = r\frac{1+\nu}{E}(p_0 - p_{\mathrm{i}}), \tag{7}$$

where ν and E are Poisson's ratio and Young's modulus of the rock mass, respectively. When the excavation advances further, p_{i} continues to decrease until eventually the decrement of stresses in the surrounding rock mass reaches a limit, σ_{re}. At this stage, plastic behavior of the rock mass in a zone with radius r_{e} surrounding the tunnel periphery starts to develop (Fig. 2.3). σ_{re} can be calculated as (Stille et al. 1989):

$$\sigma_{\mathrm{re}} = \frac{2}{1+k}(p_0 + a) - a \tag{8}$$

and r_{e} as:

$$r_{\mathrm{e}} = r\left[\frac{\sigma_{\mathrm{re}} + a}{p_{\mathrm{i}} + a}\right]^{\frac{1}{k-1}}, \tag{9}$$

in which

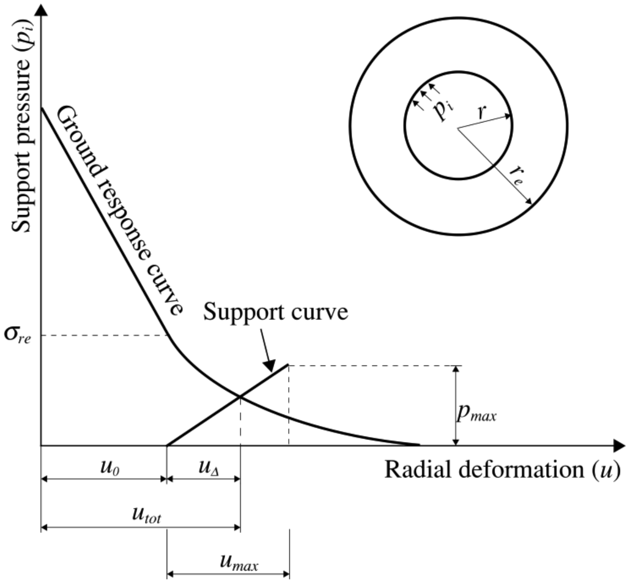

Figure 2.3. Ground and support response curves. umax is the maximum deformation that the shotcrete can withstand, u0 is the deformation that has developed when the excavation face reaches the considered cross-section, uΔ is the deformation of the shotcrete, and utot is the total expected deformation of the tunnel periphery. pmax and σre are defined in the text below. (© Bjureland et al. 2017, CC–BY 4.0, https://creativecommons.org/licenses/by/4.0).

$$k = \tan^2\left(45 + \frac{\varphi}{2}\right) \tag{10}$$

and

$$a = \frac{c}{\tan\varphi}. \tag{11}$$

where c is the cohesion of the rock mass. As soon as plastic behavior has been induced, the radial deformations of the tunnel periphery are no longer u_{ie} but instead plastic radial deformations of the tunnel periphery, u_{ip}. The u_{ip} can be calculated as:

$$u_{\mathrm{ip}} = \frac{rA}{f+1}\left[2\left[\frac{r_{\mathrm{e}}}{r}\right]^{f+1} + (f-1)\right], \tag{12}$$

where

$$A = \frac{1+\nu}{E}(p_0 - \sigma_{\mathrm{re}}) \tag{13}$$

and

$$f = \frac{\tan\left(45 + \frac{\varphi}{2}\right)}{\tan\left(45 + \frac{\varphi}{2} - \psi\right)}. \tag{14}$$

ψ is the dilatancy angle of the rock mass.

As excavation progresses passed the considered cross section, the distance x from the cross section to the excavation face increases. For small values of x, i.e. when the excavation face is close to the considered cross section, the undisturbed rock mass in front of the excavation will partly support the tunnel periphery, usually referred to as a fictitious supportive pressure that limits deformations. However, this fictitious supportive pressure decreases as the excavation progresses. Eventually, the fictitious supportive pressure does not counteract the deformation and thereby the maximum deformation, u_{final}, will be reached. The development of deformations follows a non-linear relationship (Fig. 2.4) as (Chang 1994):

$$u_{\mathrm{x}} = u_{\mathrm{final}}\left[1 - \left(1 - \frac{u_0}{u_{\mathrm{final}}}\right)\left(1 + 1.19\frac{x}{r_{\mathrm{e,max}}}\right)^{-2}\right], \tag{15}$$

in which $r_{\mathrm{e,max}}$ is the maximum radius of the plastic zone.

When the face of the excavation reaches the considered cross section, approximately one third of the final deformation that can be expected for an unsupported tunnel has developed. The following relationship can be used to approximate the magnitude of this deformation (Chang 1994):

$$u_0 = 0.279\left(\frac{r_{\mathrm{e}}}{r}\right)^{0.203 u_{\mathrm{ie}}}. \tag{16}$$

To limit deformations, different support measures can be utilized. Regularly, the support is illustrated by a separate support curve that crosses the ground–response curve at some particular deformation, i.e. the final supportive deformation. One available support measure for limiting of deformations is shotcrete. The response curve for a shotcrete support can be calculated as (Stille et al. 1989):

$$p_{\mathrm{i}} = k_c \Delta u_s, \tag{17}$$

where Δu_{s} is the deformation of the shotcrete and k_{c} is the stiffness of the shotcrete, given by

$$k_{\mathrm{c}} = \frac{E_{\mathrm{c}}}{r}\frac{r^2 - (r - t_{\mathrm{s}})^2}{(1 + \nu_{\mathrm{c}})[(1 - 2\nu_{\mathrm{c}})r^2 + (r - t_{\mathrm{s}})^2]}, \tag{18}$$

in which t_{s} is the shotcrete thickness. The relationship given in Eq. 17 is valid until the maximum pressure capacity of the shotcrete, p_{max} (Fig. 2.3) is reached. p_{max} can be calculated as

$$p_{\mathrm{max}} = \frac{1}{2}\sigma_{\mathrm{cs}}\left[1 - \frac{(r - t_{\mathrm{s}})^2}{r^2}\right], \tag{19}$$

where the σ_{cs} is the uniaxial compressive strength of the shotcrete.

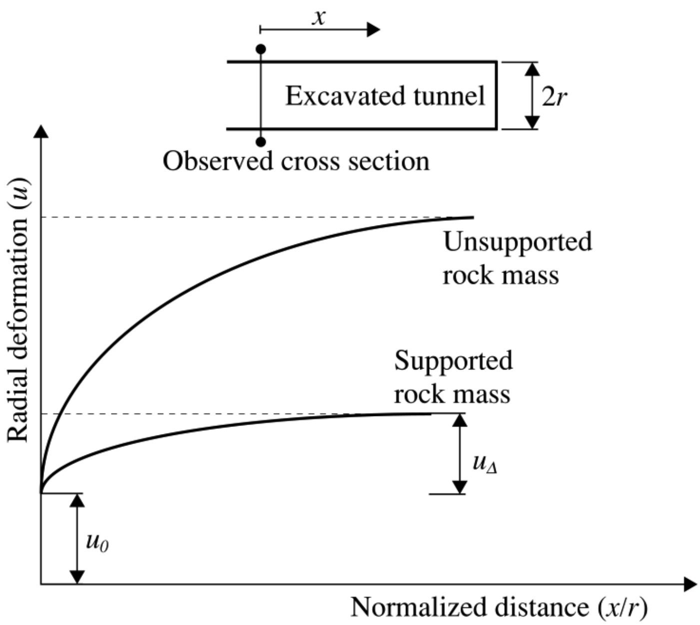

Figure 2.4. Development of deformation of the tunnel periphery during excavation for an unsupported and supported rock mass. (© Bjureland et al. 2017, CC–BY 4.0, https://creativecommons.org/licenses/by/4.0).

3. Reliability-based design methods

3.1 Factors of safety and limit state design

To account for uncertainties in design of rock tunnels, the common approach has historically been to use the deterministic total safety factor concept. The basic idea is then to ensure that the resistance of a structure is greater than the load expected to act on it by a certain margin. This margin is commonly referred to as the safety factor, SF, and is usually defined as the ratio between the mean resistance, μ_{R}, of a structure and the mean load, μ_{S}, expected to act on it:

$$SF = \frac{\mu_{\mathrm{R}}}{\mu_{\mathrm{S}}}. \tag{20}$$

By creating this margin, it is assumed that uncertainty related to load and resistance is accounted for.

The magnitude of the required SF for different limit states has in rock engineering design historically been determined heuristically, e.g. based on a long experience of similar successful, or unsuccessful, projects. This, however, has led to a situation where the required SF for a certain limit state might not, in design codes and guidelines, be calibrated against society's required levels of safety.

To overcome this, the Eurocodes (CEN 2002) applies another approach: limit state design. The preferred limit state design method according to the Eurocodes (CEN 2002) is the partial factor method.

The partial factor method's utilization in civil engineering originates from work performed in the mid-1900s by structural engineers, such as Freudenthal (1947). At that time, Freudenthal and his peers had, similary to the authors of the Eurocode, begun to question the deterministic design approach's ability to account for uncertainties present in design of structures. Instead, they began to use reliability-based methods to connect the probability of structural failure to uncertainty in load, S, and resistance, R. This led to the possibility of using reliability-based methods to account for uncertainties in design by defining a limit state function, G, as the limit between safe and unsafe behavior

$$G(\boldsymbol{X}) = 0, \tag{21}$$

in which $\boldsymbol{X}$ is a vector that contains all relevant uncertain random parameters. The probability of exceeding this limit, p_{f}, is

$$p_{\mathrm{f}} = P(G(\boldsymbol{X}) \leq 0) = \Phi(-\beta). \tag{22}$$

In its most simple form $G(\boldsymbol{X}) = R - S$. For a normally distributed $G(\boldsymbol{X})$ the corresponding reliability index, β, is defined as

$$\beta = \frac{\mu_G}{\sigma_G} \tag{23}$$

in which Φ is the cumulative standard normal distribution and μ_G and σ_G are the mean and standard deviation of G, respectively. Thus, β is a measure of the distance from the μ_G to the origin, $G(\boldsymbol{X}) = 0$, measured in σ_G (Figure 3.1).

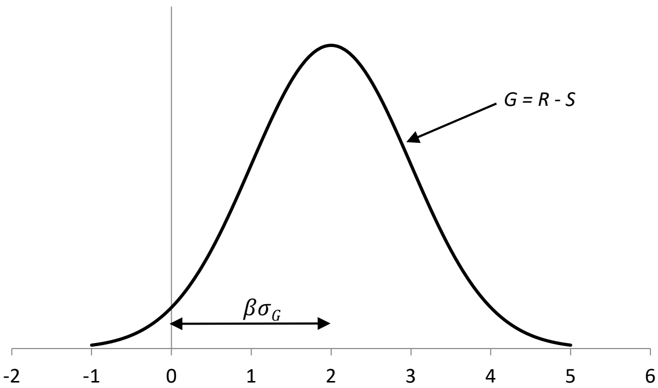

Figure 3.1. Example showing a normal distribution with $\mu_G = 2$, $\sigma_G = 1$, and consequently, $\beta = 2$.

3.2 What is failure?

Essential for the calculation of failure probability is to define what the term "failure" actually refers to. In an ultimate limit state analysis, failure often refers to attainment of the limit state, even though exceedance of the limit of that limit state does not necessarily lead to structural collapse, which is implied by the term failure. In a serviceability limit state analysis, excessive deformations can be referred to as failure, even though, similary to the ultimate limit state analysis, excessive deformations usually do not lead to structural collapse.

Herein, the term "failure" refers to exceedance of a defined limit and therefore should be read as limit exceedance. Exceedance of that limit does not necessarily cause the structure to collapse.

3.3 Frequentist and Bayesian views on probability

The term probability is in structural engineering design commonly interpreted as the long term frequency of occurrence of an event in an uncertain situation. In many situations, such an interpretation might be appropriate. However, there are situations in which it is not (Bertsekas & Tsitsiklis 2002). As an example of a situation in which it is not; consider a situation in which excavation of a rock tunnel through a well-known weakness zone is planned. The client asks the design-engineer to predict the probability that the weakness zone is water bearing and as a consequence the client wants the design engineer to judge the probability that a large ingress of water into the tunnel is to be expected. In such situations there might be information available, concerning for example the extent and permeability of the weakness zone, but not in terms of frequencies. From a frequentist point of view, this information therefore becomes irrelevant, since excavation through the weakness zone in this particular location is a onetime event. In rock tunnel engineering, the design engineer often has to make decisions in such situations. Therefore, to assign and use subjective degrees of beliefs in the design process of rock tunnels is preferable.

Using subjective degrees of beliefs is the core of the Bayesian interpretation of probability. In the Bayesian interpretation, all uncertainties are described and accounted for as accurately as possible, based on the information available to the designer. The Bayesian interpretation is, in that sense, wider than the frequentist interpretation, because it allows for incorporation of both objective data and subjective degrees of beliefs in the analysis (Vrouwendeler 2002, Johansson et al. 2016).

In practice, another relatively common interpretation of probability is the nominal one. In the nominal interpretation, it is acknowledged that some approximations and simplifications have been made in the calculated probability and that some known uncertainties are left unaccounted for. When these issues are ignored the calculated probability has no connection to the reliability of the structure, i.e. the calculated probability becomes nominal (Melchers 1999). However, even if the calculated probability becomes nominal it can, if calibrated, be used as a basis for decision making.

As argued for by other authors (Vrouwendeler 2002, e.g. Baecher & Christian 2003, Johansson et al. 2016) the Bayesian interpretation is the most useful interpretation of probability. Compared to the nominal interpretation the Bayesian interpretation requires that all uncertainties are described and accounted for as accurately as possible, based on the information available to the designer. For this reason, the Bayesian view on probability is used herein and thereby the term probability should be interpreted as degree of belief.

3.4 Acceptable probability of failure

When using reliability-based methods, it must be shown that the designed structure fulfills the levels of safety required by society. In the Eurocode (CEN 2002), society's demands on acceptable levels of safety in ultimate limit states are defined as a target reliability index, β_{target}, or as a target probability of failure, $p_{\text{f,target}}$, with a magnitude that depends on the reliability class of the structure. The β_{target} and $p_{\text{f,target}}$ values that must be achieved for individual components of a structure can be seen in Table 3.1. The reliability class of the structure is in turn related to the consequences of limit state attainment. Similarly to reliability classes, the Eurocode (CEN 2002) therefore divides this into three different levels. The consequence classes can be seen in Table 3.2. Most rock tunnels belong to reliability and consequence class 3.

Table 3.1. Acceptable levels of safety according to Eurocode.

Reliability class	β_{target}	$p_{\text{f,target}}$
RC1	4.20	$1.33 * 10^{-5}$
RC2	4.70	$1.30 * 10^{-6}$
RC3	5.20	$1.00 * 10^{-7}$

Table 3.2. Definition of consequence classes in Eurocode.

Consequence class	Description	Example
CC1	Small risk of death, and small or negligible economical, societal or environmental consequences.	Farm buildings where people don't normally reside.

CC2	Normal risk of death, considerable economical, societal or environmental consequences.	Residence and office buildings.
CC3	Large risk of death, or very large economical, societal or environmental consequences.	Stadium stands and concert halls.

3.5 Uncertainties

3.5.1 Categorization of uncertainties

A common feature in design and construction of rock tunnels is that large uncertainties, are present. Generally, these uncertainties are divided into two broadly defined categories: aleatory and epistemic uncertainties. In rock engineering, aleatory uncertainty is due to the inherent variability, or randomness, in input parameters and cannot be reduced. Epistemic uncertainty on the other hand is uncertainty that is due to a lack of knowledge and can therefore be reduced as more knowledge is gained (Ang & Tang 2007). Uncertainties present in rock engineering are mainly epistemic.

An alternative way of categorizing uncertainties is to do so based on their sources. Baecher & Christian (2003) divided uncertainties into three categories: characterization uncertainty, model uncertainty, and parameter uncertainty. Characterization uncertainty is related to uncertainty in the interpretation results from site investigations. Model uncertainty relates to uncertainty in the applied calculation model. Parameter uncertainty relates to the uncertainty that might be introduced in the operationalization of a measurement, i.e. the transformation from an observed parameter to an inferred property of interest.

Similarly, Melchers (1999) also categorized uncertainties based on their sources, but argued that there are seven main sources: phenomenological uncertainty, decision uncertainty, modelling uncertainty, prediction uncertainty, physical uncertainty, statistical uncertainty, and uncertainty due to human factors. A description for each source of uncertainty can be found in Table 3.3.

Following the categorization made by Baecher & Christian (2003), characterization, model, and parameter uncertainty are all present in design and construction of rock tunnels. Taking the limit states presented in Sections 2.2 and 2.3 as an example, characterization and parameter uncertainty are incorporated through the input parameters. Model uncertainty is incorporated through the use of the presented analytical calculations, which are based on simplifying assumptions.

Table 3.3. Description of the sources of uncertainty (Melchers 1999).

Source of uncertainty	Description
Phenomenological	Uncertainty in the phenomena relevant for a structure's expected behavior.
Decision	Decision of whether or not a particular phenomenon has occurred.
Modelling	Uncertainty in the applied calculation model, i.e. how well the model represents the physical behavior of the physical structure.
Prediction	Concerns our ability to predict the future behavior of a structure, e.g. the prediction of expected deformations when a structure is being exposed to loads.
Physical	Relates to the inherent variability, or randomness, of the basic variables.
Statistical	Concerns the determination of statistical estimators to suggest an appropriate probability density function.
Human errors	Due to the natural variation in task performance and those which occur in the process of design, documentation, and construction and use of the structure within accepted processes. In addition, uncertainties due to gross human errors are those which are a direct result of neglecting fundamental structural or service requirements.

3.5.2 Spatial variability of input parameters

Depending on the source of an uncertainty, different approaches to account for it are suitable. When using reliability-based methods, characterization and parameter uncertainties can be accounted for by quantifying a parameter in terms of its spatial variability, describe it in terms of a suitable probability distribution, and incorporate the quantified parameter in a reliability-based calculation.

This can be done using the mean, μ, standard deviation, σ, and the scale of fluctuation, θ, of the parameter. The θ is a measure of the distance in space within which the magnitude of a parameter shows strong correlation with itself (Vanmarcke 1977). It is commonly estimated by fitting a theoretical correlation function, $\rho(\tau)$, to a set of data for the parameter of interest (e.g. Lloret-Cabot et al. 2014). In such a case, the θ defines the correlation between two points in space separated by a distance τ. An example of a common correlation function is the Gaussian, which for the correlation between two points in direction *z* is expressed as (Shi & Stewart 2015):

$$\rho(\tau_z) = \exp\left(-\pi\left(\frac{|\tau_z|}{\theta_z}\right)^2\right), \tag{24}$$

in which $\tau_z = z_i - z_j$ is the distance between the two points i and j in direction z and θ_z is the scale of fluctuation in direction z.

By knowing the θ, the σ of the parameter of interest can be reduced using variance reduction techniques, because the variance reduction factor, Γ, depends on θ in relation to the geometrical size, Δ, of the studied domain. For a parameter with equal θ in two directions, x and y, and equal Δ in the same directions, i.e. $\Delta x = \Delta y = \Delta$, the Γ can be calculated as (Vanmarcke 1977):

$$\Gamma(\Delta x, \Delta y) = \left(\frac{\theta_x \theta_y}{\Delta x \Delta y}\right)^{\frac{1}{2}} = \frac{\theta}{\Delta}, \tag{25}$$

where θ_x and θ_y are the scale of fluctuations in the x and y directions, respectively. Note that Eq. 5 is only valid for $\theta \leq \Delta$. If $\theta \geq \Delta$, then $\Gamma = 1$. The effect of Γ on the σ of the mean value of the parameter of interest is:

$$\sigma_r = \Gamma\sigma, \tag{26}$$

where σ_r is the reduced standard deviation.

The mean and standard deviation along with suggestions on suitable probability distributions for the input parameters to the limit states for shotcrete presented in Section 2.2 are quantified in Bjureland et al. (2019). The effect that spatial correlation has on the presented standard deviation of shotcrete thickness is discussed in Bjureland et al. (2019) and the effect that spatial variability has on the load-bearing capacity of shotcrete is discussed in Bjureland et al. (2020).

3.5.3 Uncertainty related to our calculation models

To account for uncertainty in calculation models and the effect that the potential model error has on the analyzed limit state, the results obtained when using the calculation model must be compared with those obtained from the structure when it is exposed to that same limit state. For the limit states presented in Sections 2.2. and 2.3, such a comparison is not feasible. A possibility for these cases is therefore to simulate reality using numerical simulations.

Model uncertainty introduced through the assumption that the structural behavior in the limit states in Section 2.2 is governed by the mean value of its input parameters and neglecting the effect that block stiffness has on the load-bearing capacity of shotcrete, is analyzed and discussed in Bjureland et al. (2020).

3.6 Methods for reliability-based design calculations

3.6.1 General reliability theory

In a general case, Eq. 22 can be solved by evaluating the following multidimensional integral over the unsafe region (Melchers 1999):

$$p_{\mathrm{f}} = P[G(\boldsymbol{X}) \leq 0] = \int \dots \int_{\mathrm{G}(\mathbf{X})\leq 0} f_{\mathbf{X}}(\boldsymbol{x}) d\boldsymbol{x}, \quad (27)$$

in which $f_{\mathbf{X}}(\boldsymbol{x})$ is a joint probability density function that describes all random variables. This integral is in most cases very difficult, or even impossible, to solve analytically. Therefore, several methods that approximate the integral have been developed. These methods are usually divided into three, or four, different levels based on their approach of accounting for uncertainties in input variables. The following categorization of the different approaches can be made (Melchers 1999):

- Level I methods account for uncertainty by adding partial factors or load and resistance factors to characteristic values of individual uncertain input variables. Two examples are the partial factor method and the load and resistance factor design.
- Level II methods account for uncertainty through the mean, μ, standard deviation, σ, and correlation coefficients, ρ, of the uncertain random input variables. However, the methods assume normal distributions. Examples here are simplified reliability index and second-moment methods.
- Level III methods account for uncertainty by considering the joint distribution function of all random parameters. One example of a Level III method is Monte Carlo simulations.
- Level IV methods add the consequences of failure into the analysis, thereby providing a tool for, e.g., cost-benefit analyses.

As the Level IV includes consequences, it is sometimes excluded in the categorization of the different methods.

3.6.2 The partial factor method

The preferred design method in the Eurocodes (CEN 2002) is the partial factor method, even though the Eurocodes' version is slightly adjusted from the original method. The original partial factor method is a limit state design method that accounts for uncertainties by applying a partial factor to the characteristic values of S and R.

In the original version of the method, partial factors have a clear connection to reliability-based design and they are statistically derived for both S and R from the general expressions (Melchers 1999)

$$\gamma_{\mathrm{S,j}} = \frac{x_{\mathrm{d},j}}{x_{\mathrm{k},j}} = \frac{F_{\mathrm{X}_j}^{-1}\left[\Phi(y_j^*)\right]}{x_{\mathrm{k},j}} \tag{28}$$

and

$$\gamma_{\mathrm{R},i} = \frac{x_{\mathrm{k},i}}{x_{\mathrm{d},i}} = \frac{x_{\mathrm{k},i}}{F_{\mathrm{X}_i}^{-1}[\Phi(y_i^*)]}, \tag{29}$$

respectively, in which $x_{\mathrm{k},i}$ and $x_{\mathrm{k},j}$ represents characteristic values of a particular uncertain parameter; $x_{\mathrm{d},i}$ and $x_{\mathrm{d},j}$ are design values of that same parameter that can be found by transforming the coordinates of Hasofer and Lind's (1974) design point, $\boldsymbol{y}^*$, back from standard normal space, Y. This back transformation is denoted $F_{\mathrm{X}_j}^{-1}\left[\Phi(y_j^*)\right]$ and $F_{\mathrm{X}_i}^{-1}[\Phi(y_i^*)]$, respectively, in Eq. 28 and Eq. 29. Principally, $x_{\mathrm{d},i}$ and $x_{\mathrm{d},j}$ are dependent on the variable's μ the directional cosines (sensitivity factors), α_i, the β_{target}, and the coefficient of variation, COV. Extended presentations of α_i and β_{target}, are given in Sections 3.6.3 and 3.4, respectively.

As previously mentioned, in the Eurocodes version of the partial factor method, fixed partial factors are proposed for different materials. The proposed values are mainly based on two factors: a long experience of building tradition (the most common approach in Eurocode), and on the basis of statistical evaluation of experimental data and field observations (CEN 2002).

3.6.3 Second-moment and transformation methods

Second-moment methods started to gain recognition in the late 1960s, based essentially on the work performed by Cornell (1969). The second-moment methods belong to a group of approximate methods that can be used to calculate p_{f} by approximating the integral in Eq. 27 through the first two moments in the random variables, i.e. the μ and σ. However, generally, the $G(\boldsymbol{X})$ is not linear and thereby the first two moments of $G(\boldsymbol{X})$ are not available (Melchers 1999). To solve this, the second-moment methods uses Taylor series expansion about some point, x^*, to linearize $G(\boldsymbol{X})$. Approximations that linearize $G(\boldsymbol{X})$

are usually referred to as "first order" methods (Melchers 1999). To solve this, the second-moment methods use Taylor series expansion about some point, x^*, to linearize G(X). Approximations that linearize G(X) are usually referred to as "first-order" methods (Melchers 1999).

In the early 1970s, an improvement to this approach was proposed by Hasofer & Lind (1974). By transforming all variables to their standardized form, standard normal distribution, $N(0,1)$, computation of β becomes independent of algebraic reformulation of $G(\boldsymbol{X})$. This method is usually referred to as the "first-order reliability method" (FORM). Further improvements have since then been made for situations such as for non-normal distributions and for correlation between variables (e.g. Hochenbichler & Rackwitz 1981).

In principle the methodology used in FORM is as follows. First, all random variables and the limit state function are transformed into Y through:

$$Y_i = \frac{\mathrm{X}_i - \mu_{X_i}}{\sigma_{X_i}}, \tag{30}$$

in which Y_i is the transformed variable, X_i, with $\mu_{Y_i} = 0$ and $\sigma_{Y_i} = 1$. The μ_{X_i} and σ_{X_i} are the mean and standard deviation of the X_i, respectively (Melchers 1999).

In the Y, the $G(\boldsymbol{Y})$ is a linearized hyperplane from which evaluation of the shortest distance to the origin yields β. This evaluation can be made through:

$$\beta = \min_{\mathrm{G}(\mathbf{Y})=0} \sqrt{\sum_{i=1}^{n} y_i^2}, \tag{31}$$

in which y_i represents the coordinates of any point on the limit state surface, $G(\boldsymbol{Y})$ (Melchers 1999). The point that is closest to the origin is often referred to as the "design point" or "checking point", y^*, and it represents the point of greatest probability for the $g(\boldsymbol{Y}) < 0$ domain.

One very useful feature of FORM is that α_i can be derived. The α_i can be found by first calculating the outward normal vector, c_i, to the $g(\boldsymbol{Y}) = 0$

$$c_i = \lambda \frac{\partial \mathrm{g}}{\partial \mathrm{y}_i}, \tag{32}$$

in which λ is an arbitrary constant, and then calculating the length of the outward normal vector, l,

$$l = \sqrt{\sum_i c_i^2}. \tag{33}$$

The α_i is defined as

$$\alpha_i = \frac{c_i}{l} \tag{34}$$

and indicates how sensitive $G(\boldsymbol{Y})$ is to changes in the respective Y_i.

3.6.4 Monte Carlo simulations

Monte Carlo simulations are a repetitive numerical process for calculating probability (Ang & Tang 2007). The process starts with generating a random number from the assigned probability density function of each of the predefined random variables, $\hat{x}$. For each repetition, $G(\hat{\boldsymbol{x}})$ is evaluated and for every combination of $\hat{\boldsymbol{x}}$ where $G(\hat{\boldsymbol{x}}) \leq 0$, the limit between the safe and unsafe behavior, defined by G, is exceeded; i.e. the result is deemed as "failure". Repeating the process for a large number of repetitions, counting the number of "failures", and comparing them with the total number of repetitions, N, gives an estimate of p_f.

The accuracy of the calculated p_f is dependent on N and the magnitude of the calculated p_f. In principle, the smaller p_f is the larger N must be to gain the same level of accuracy of the calculated p_f. To find the required number of calculations to achieve a particular level of accuracy, the following can be used (Harr 1987). As each simulation is an experiment with a probability of a successful result, p_s, and a probability of an unsuccessful result, p_u, equal to $1 - p_\mathrm{s}$, assuming that the simulations are independent. Thus, the simulations will yield a binomial distribution with an expected value of Np_s and a standard deviation of $\sqrt{Np_\mathrm{s}(1 - p_\mathrm{s})}$. Then if x_su (which will be normally distributed) is defined as the number of successes in N simulations and $x_{\tilde{\alpha}/2}$ as the number of successes in N simulations such that the probability of a value larger or smaller, then that value is not greater than $\tilde{\alpha}/2$, the number of simulations required, N_{req}, is

$$N_{req} = \frac{p_\mathrm{s}(1 - p_\mathrm{s})h_{\tilde{\alpha}/2}^2}{e^2}, \tag{35}$$

in which $h_{\tilde{\alpha}/2}$ is the normally distributed quantile for a chosen credibility level and e represents the maximum allowable system error given as

$$e = p_\mathrm{s} - \left(\frac{x_{\frac{\tilde{\alpha}}{2}}}{N}\right). \tag{36}$$

As can be seen from Eq. 35, $p_s(1-p_s)$ is maximized when p_s is ½. Thereby, a conservative approach is to use $p_s(1-p_s) = 1/4$, which, for a limit state with a single variable, yields that

$$N_{req} = \frac{h_{\tilde{\alpha}/2}^2}{4e^2} \quad (37)$$

and for a limit state with multiple variables, *m*,

$$N_{req} = \left(\frac{h_{\tilde{\alpha}/2}^2}{4e^2}\right)^m. \quad (38)$$

3.7 Conditional probability and Bayes' rule

Many limit states in a reliability-based analysis are conditioned on the occurrence of a particular event, such as the exceedance of another limit state. According to Bayes' rule, the probability of an event A_i, occurring given that an event *B* has occurred, is (Bertsekas & Tsitsiklis 2002)

$$\mathrm{P}(A_i|B) = \frac{\mathrm{P}(A_i)\mathrm{P}(B|A_i)}{\mathrm{P}(B)} = \frac{\mathrm{P}(A_i)\mathrm{P}(B|A_i)}{\mathrm{P}(A_1)\mathrm{P}(B|A_1) + \cdots + \mathrm{P}(A_n)\mathrm{P}(B|A_n)}, \quad (39)$$

in which $\mathrm{P}(B|A_i)$ is the probability of event *B* occurring conditioned on the fact that event A_i has occurred; which in turn can be found through the conditional,

$$\mathrm{P}(B|A_i) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}, \quad (40)$$

and total probability theorems

$$\mathrm{P}(B) = \mathrm{P}(A_1 \cap B) + \cdots + \mathrm{P}(A_n \cap B) = \mathrm{P}(A_1)\mathrm{P}(B|A_1) + \cdots + \mathrm{P}(A_n)\mathrm{P}(B|A_n). \quad (41)$$

An illustration of how conditional probability and Bayes' rule can be utilized in design of shotcrete support is presented in Bjureland et al. (2019), in Appendix 1, and Bjureland et al. (2017). In Bjureland et al. (2019) and in Bjureland et al. (2020), conditional probability is used to evaluate the probability of exceeding the shotcrete's capacity. In Appendix 1, Bayes' rule is used to update the magnitude and uncertainty of shotcrete thickness as control measurements from laser scanning is obtained. In Bjureland et al.

(2017), Bayes' rule is used to update the probability of limit exceedance after measurements of deformations have been performed.

3.8 System reliability

When determining the capacity of rock tunnel support, it is fairly common that the joint effect of multiple limit states must be considered, that is the support must be analyzed as a structural system. The probability of exceeding the capacity of such a structural system can be found by evaluating the multidimensional integral in Eq. 27 but overall the unsafe regions, D_i, as (Melchers 1999):

$$p_\mathrm{f} = P[G_i(\boldsymbol{X}) \leq 0] = \int_{\cup \cap D_i \in X} \dots \int f_\mathbf{X}(\boldsymbol{X}) d\boldsymbol{x}. \tag{42}$$

Generally, structural systems are idealized into two main types: series and parallel systems. In a series system, failure of the entire system is obtained when the limit state for the weakest component occurs. Series systems are usually referred to as weakest link systems and are typified by a chain (Melchers 1999). The system failure probability for a series system of i components is (Freudenthal 1962, Freudenthal et al. 1964):

$$p_\mathrm{f} = P(\cup A_i) = P(\cup \{G_i(\boldsymbol{X}) \leq 0\}). \tag{43}$$

In a parallel system, usually referred to as a redundant system, failure of one component do not necessarily cause the entire system to fail. Instead, failure of a parallel system is obtained when the limit state for all its contributory components occurs. The system failure probability for a parallel system of i components is (Melchers 1999):

$$p_\mathrm{f} = P(\cap A_i) = P(\cap \{G_i(\boldsymbol{X}) \leq 0\}). \tag{44}$$

In parallel systems with elastic-brittle components and low redundancy, failure of one component is followed by failure of the entire system since the redistribution of loads causes excessive loading of other components (Melchers 1999). In such systems, it is therefore commonly assumed that failure of the component exposed to the highest load, with respect to its capacity, leads to failure of the entire system. For redundant parallel systems with elastic-plastic components or components with residual strength, the opposite is true. Such systems commonly act as "true" parallel systems with successful redistribution of loads in between individual components of the system.

In reality, structural systems will commonly consist of subsystems of the two main types and some systems contain conditional aspects in which failure of one component affects the probability of failure in another component in the same system (Melchers 1999). In addition, in some systems two or more components might be correlated. In such cases, the correlation must be accounted for in the calculation of p_f, which can be a complicated task. An example of the effect of correlation on the system pf for a parallel system is illustrated in Figure 3.2.

An effective approach to dealing with both conditional aspects and correlation between components in a structural system is to use Monte Carlo simulations. By doing so, both the conditional aspects and the correlation between components can be accounted for directly in the simulations.

Another approach is to use bounds. The p_f is then expressed using an upper and a lower bound, with the "correct" p_f being somewhere in-between. The first order bounds for a series system of *i* components are given by (Melchers 1999):

$$\max_{i=1}^{n} P(A_i) \leq p_\mathrm{f} \leq 1 - \prod_{i=1}^{n} (1 - P(A_i)) \tag{45}$$

and for a parallel system, the bounds are given by:

$$1 - \prod_{i=1}^{n} P(A_i) \leq p_\mathrm{f} \leq \min_{i,j=1}^{n} P(A_i) \quad \text{for } \rho_{i,j} > 0, \tag{46}$$

in which $\rho_{i,j}$ is the correlation coefficient of between components i and j. A drawback with bounds, however, is that they are so wide that they are rarely useful in practical applications (Grimmelt & Schuëller 1982).

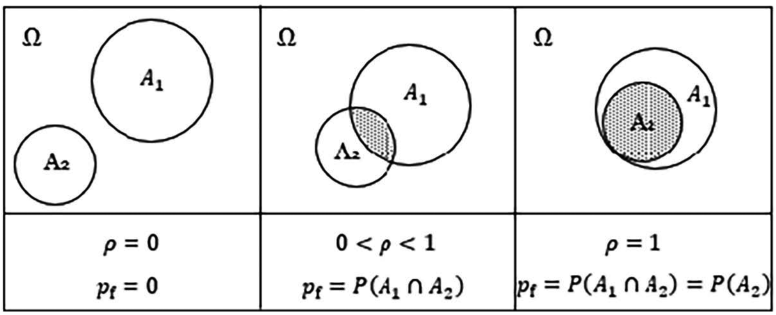

Figure 3.2. Venn diagram illustrating the probability of failure, p_f, for a two component, A_1 and A_2, parallel system in the sample space Ω with different correlations, ρ (modified after Krounis (2016)).

In Bjureland et al. (2019) and Appendix 1, the system p_f for shotcrete support is calculated and both conditional aspects and correlation between limit states are considered. The structural subsystem behavior of shotcrete support when exposed to bending moments is analyzed and discussed in Bjureland et al. (2020).

4. The observational method

An alternative, or complementary, accepted limit state verification method in Eurocode 7 (CEN 2004) that can be used to account for uncertainties, is the observational method. The observational method is usually credited to originate from the work by Terzaghi and Peck in the early and mid-1900s , even though successful similar approaches had been used before (e.g. the final report by the Geotechnical Committee of the Swedish State Railways (1922)). The main idea of the observational method is to predict the behavior of a geotechnical structure in an initial design, observe the behavior of the structure during construction, and verify that the structure's behavior fulfills formulated requirements. The method is similar to the, at least in Sweden, well-known approach called "active design" (Stille 1986).

4.1 The observational method according to Terzaghi and Peck

One of the key considerations of Terzaghi's and Peck's and formulation of the observational method was to account for uncertainties, for safety and optimization reasons, in design of underground excavations. In line with these considerations, Peck (1969) defined a number of elements that must be included in a complete application of the method:

a. "Exploration sufficient to establish at least the general nature, pattern and properties of the deposits, but not necessarily in detail.
b. Assessment of the most probable conditions and the most unfavourable conceivable deviations from these conditions. In this assessment geology often plays a major role.
c. Establishment of the design based on a working hypothesis of behaviour anticipated under the most probable conditions.
d. Selection of quantities to be observed as construction proceeds and calculation of their anticipated values on the basis of the working hypothesis.
e. Calculations of values of the same quantities under the most unfavourable conditions with the available data concerning the subsurface conditions.
f. Selection in advance of a course of action or modification of design for every foreseeable significant deviation of the observational findings from those predicted on the basis of the working hypothesis.
g. Measurement of quantities to be observed and evaluation of actual conditions.
h. Modification of design to suit actual conditions."

4.2 The observational method as defined in Eurocode 7

Similar to Peck's suggestion, Eurocode postulates that certain principles are satisfied for a successful application of the methodology. These principles are comparable to the elements included in Peck's suggestion, but are defined slightly different:

"(1) When prediction of geotechnical behavior is difficult, it can be appropriate to apply the approach known as 'the observational method' in which the design is reviewed during construction.

(2) P The following requirements shall be met before construction is started:

a) acceptable limits of behavior shall be established;
b) the range of possible behavior shall be assessed and it shall be shown that there is an acceptable probability that the actual behavior will be within the acceptable limits;
c) a plan of monitoring shall be devised, which will reveal whether the actual behavior lies within the acceptable limits. The monitoring shall make this clear at a sufficiently early stage, and with sufficiently short intervals to allow contingency actions to be undertaken successfully;
d) the response time of instruments and the procedures for analyzing the results shall be sufficiently rapid in relation to the possible evolution of the system;
e) a plan of contingency actions shall be devised, which may be adopted if the monitoring reveals behavior outside acceptable limits.

(3)P During construction, the monitoring shall be carried out as planned.

(4)P The results of the monitoring shall be assessed at appropriate stages and the planned contingency actions shall be put into operation if the limits of behavior are exceeded.

(5)P Monitoring equipment shall either be replaced or extended if it fails to supply reliable data of appropriate type or in sufficient quantity." (CEN 2004)

The principles marked with "P" must not be violated.

4.3 The use of the observational method in today's tunneling

Even though underground construction was one of the key considerations for the formulation of the observational method, the methodology, as defined in Eurocode 7, for design of rock tunnels is rarely utilized in practice. One reason for this, as argued for by Spross (2016), might be that the inflexible requirements, such as showing that the geotechnical behavior with a sufficient probability will be within the acceptable limits, reduce the applicability of the method. In addition, the lack of guidance on how the requirements can be fulfilled hampers the implementation further.

To increase its applicability, Spross (2016), like other authors (e.g. Palmstrom & Stille 2007, Maidl et al. 2011, Zetterlund et al. 2011), suggests that reliability-based methods should be incorporated into the framework of the observational method. By doing so, the reliability-based methods can be used (Spross et al. 2014, Holmberg & Stille 2007, 2009, Stille et al. 2005, Spross & Johansson 2017) to perform a preliminary design in which a prediction is made about the structure's most probable and possible behavior.

5. The use of reliability-based methods in design of rock tunnel support

The utilization of reliability-based design methods in rock tunnel engineering has to some extent been addressed earlier. One of the early contributors to the subject was Kohno (1989). Kohno performed relatively extensive work over a large span of areas covering topics of both type I, that is limit states with separable load and resistance, and type II, that is limit states with interaction between the load and the resistance, such as reliability of tunnel support in soft rock, reliability of tunnel lining in jointed hard rock, probabilistic evaluation of tunnel lining deformation through observation, and reliability of systems in tunnel engineering. Other contributions to the field have mainly been limited to the analysis of limit states of either type I or type II. This chapter reviews some of the performed work in both types of limit states.

5.1 Limit states with separable load and resistance

For limit states of type I, research has mainly been performed on the analysis of rock wedges, both in slopes and tunnels. As an example, Quek & Leung (1995) analyzed the reliability of a rock slope using the first-order second-moment method, complementing it with Monte Carlo simulations. On the same subject, Low (1997) analyzed sliding stability of a rock wedge in a rock slope. Low used an Excel spreadsheet and second-moment reliability indices with both single and multiple failure modes to calculate the probability of sliding failure of a rock wedge. Similarly to the work performed by Low (1997), Jimenez-Rodriguez et al. (2006) and Jimenez-Rodriguez & Sitar (2007) analyzed the stability of a rock slope and rock wedge using FORM and Monte Carlo simulations, respectively, in a system reliability analysis. The analyses showed that the results from Monte Carlo simulations could be approximated using FORM and that bounds of the probability of failure provide accurate estimations of the system probability when information regarding both uni-component and joint bi-component probabilities are considered.

To study how clamping forces, the half-apical angle, and other parameters affect the calculated partial factors and results of a stability analysis, Bagheri (2011) used both deterministic and reliability-based methods. The results show that partial factors needed for a safe design are (very) sensitive to the half-apical angle and that they change significantly from case to case. Similar results are presented in Bjureland et al. (2017). Park et al. (2012) combined deterministic calculations and reliability-based methods to derive an equation for the SF of rock wedge failure in a slope and combined it with the point estimate method to calculate the probability of limit exceedance. Further, Low & Einstein (2013) compared the results from a reliability analysis of tunnel roof wedges and forces in rock bolts, using mainly FORM and second-order reliability method (SORM) against deterministic calculations and Monte Carlo simulations.

As can be seen from the previously performed research on reliability-based methods for limit states of type I they have been used successfully for a number of different limit states. However, only a limited amount of research that concerns the design of shotcrete support for small loose rock that can fit between rockbolts has been found. In particular, no research that describes the structural system of shotcrete support and how this system can be analyzed using reliability-based methods has been found. In addition, the magnitude and uncertainty of input variables are in many of the studied cases assumed due to a lack of supporting data. Lastly, when using reliability-based methods in design of shotcrete support, it is commonly assumed that the support's capacity is an averaging system. However, research that confirms this assumptions and research that studies the influence from block stiffness has not been found.

5.2 Limit states with interaction between load and resistance

For limit states of type II, Laso et al. (1995) studied the probability of limit exceedance for tunnel support using the convergence–confinement method with four limit definitions: 1) excessive support lining pressure, 2) soil deformation, 3) lining deformation, and 4) critical strain of lining.

Celestino et al. (2006) used load and resistance factor design for design of shotcrete support with respect to bearing capacity of the support footing for the shotcrete arch and failure of the shotcrete lining support. The analysis was performed based on a case study of a tunnel in Brazil.

Similarly to the work that Bagheri (2011) and Park et al. (2012) performed for rock wedges, Nomikos & Sofianos (2010) developed an approach to use the SF in a probabilistic way. The developed method was utilized in two design situations: stability of rock pillars and stability of underground roofs in a layered rock mass.

As for rock wedges, Low has performed extensive research on limit states of type II. Firstly, Li & Low (2010) used FORM and the convergence-confinement method to perform a reliability-based analysis with two limit criteria, one for the rock mass and one for the shotcrete support lining. Secondly, Lü & Low (2011) performed a similar analysis but used SORM and the response surface method instead of FORM. Both results were compared to results from Monte Carlo simulations. Lastly, Lü et al. (2011) extended the previously performed analysis with a third limit criterion: a requirement that the length of the rock bolt must exceed the radius of the plastic zone minus the radius of the tunnel with at least 1.5 m. Similarly to Lü & Low (2011), the response surface method was used.

Zhang & Goh (2012) used empirical relationships and RMR ratings to estimate parameters for a numerical analysis of a rock cavern in the numerical modeling software

FLAC3D. Using a two variable factorial design approach in the analysis, frequency distributions for SF and strain were obtained. Based on the results, a regression model that could be used to calculate the probability of limit exceedance in a tunnel was developed. In the analysis, both ultimate and serviceability limit states were considered. Goh & Zhang (2012) used artificial neural networks (ANN) combined with FLAC3D to study the SF in a tunnel and Langford & Diederichs (2013) used numerical analysis combined with a modified point estimate method to analyze and discuss shotcrete support design.

Similar to the research performed for limit states of type I, reliability-based methods have been successfully used for a number of different limit states; however, the previously performed research mainly concerns the behavior and design of the final support. The behavior of the tunnel during construction and the design of the preliminary or temporary support, or the final support during construction, have only been studied to a limited extent.

5.3 Reliability-based methods and the observational method in rock engineering

Spross et al. 2016, Spross & Larsson 2014, Spross & Johansson 2017). Spross (2016) PhD thesis covers a wide spectrum of applications such as groundwater leakage control in tunnels, pore-pressure measurements in safety assessments of dams, and pillar stability in underground caverns. The main contribution from Spross (2016) is the presentation of a probabilistic framework for the observational method, which combines reliability-based design with Bayesian statistical decision theory. Other contributors to reliability-based design and the observational method include e.g. Stille et al. (2005), Holmberg & Stille (2007), Holmberg & Stille (2009), and Zetterlund et al. (2011). Bjureland et al. (2017) is to some extent a continuation of the previously performed research on the observational method.

6. Discussion

6.1 The applicability of the partial factor method in rock engineering design

Using the partial factor method with the fixed partial factors proposed by the Eurocodes (CEN 2002) in design of shotcrete support, and even rock tunnel support in general, presents some challenges. As illustrated in Bjureland et al. (2017), the magnitude of derived partial factors can vary significantly with a change in the geometry, or scale, of an analyzed problem since the relationship between the load and the resistance changes. The reliability level of the structure thereby changes with a change in geometry of the analyzed limit state when fixed partial factors are used. Thus, different levels of safety are achieved for different geometric layouts of the same limit state. This implies that the fixed partial factors do not work as intended in the original version of the method, described in Section 2.6.2. The original version of the partial factor method has, to some extent, the potential to stringently account for uncertainty in input parameters, sensitivity of the structural system to these parameters, and also the target reliability. The original version of the partial factor method can therefore theoretically be applied in design by calculation for limit states in which a distinction, after simplifications, can be made between the parameters affecting the load and the parameters affecting the resistance. This requires, however, that the partial factors are calibrated for every design situation. In many cases, doing so requires more effort than simply using reliability methods such as Monte Carlo simulations directly. In addition, when using the partial factor method it can be difficult to stringently account for the epistemic uncertainties that are commonly present in design and construction of rock tunnels. For these reasons, I am of the opinion that the partial factor method is unsuitable for limit states of type I.

For limit states of type II, such as the one presented in Bjureland et al. (2017), a clear distinction cannot be made between the parameters affecting the load and the parameters affecting the resistance. The partial factor method is therefore not an applicable method for these types of limit states.

6.2 On reliability-based methods and the observational method for design of shotcrete support in tunnels

Reliability-based methods can be used to overcome some of the challenges presented for the partial factor method. As can be inferred from the design methods presented in Appendix 1 and Bjureland et al. (2017), for limit states of both type I and type II, reliability-based methods are applicable in design of shotcrete support, especially when they are combined with observations or when they are used within the framework of the observational method. By applying reliability-based methods, uncertainties in input parameters, the sensitivity of the structural system to these parameters, and also the target reliability can stringently be accounted for. Depending on the type of analyzed limit state, different reliability-based methods are suitable to use for the analysis. For simple limit states of type I with linear limit state functions, simplified methods such as FORM can theoretically be used. For more complex non-linear limit states or for structural systems with multiple correlated and conditional limit states with complex distributions of input parameters, as in Bjureland et al. (2019) and Appendix 1 and Bjureland et al. (2017), Monte Carlo simulations are preferable.

6.2.1 Quantifying and accounting for uncertainties in input parameters during the design process

A challenge for the applicability of reliability-based methods in design of rock tunnel support is that a large amount of data for all relevant uncertain input parameters are required to assign suitable probability distributions to them. For parameters related to the quality of the rock mass, the large amount of data can be difficult to obtain in a single project, due to the extent of pre-investigations that would be required. For parameters related to the tunnel support, the data can be obtained through the standardized control measurements performed during construction, as exemplified in Bjureland et al. (2019). As illustrated in that paper, most of the parameters used for design of shotcrete support against small loose blocks at the Project Stockholm City Line could be quantified based on the information obtained from the control measurements performed. The amount of data largely exceeded the amount of data required for quantifying and defining the variability of the parameters.

However, a drawback of relying on the standardized control measurements to obtain data is that the data are unavailable to the design engineer until construction has been started. Thus, for both parameters related to the rock mass and parameters related to tunnel support, the use of experience gained from previously executed projects and knowledge obtained from the literature are required in the initial phase of the design process. To stringently account for the incorporated uncertainties the initial design of the tunnel support must be complemented with continuously performed observations during construction. And that the final tunnel support's capacity must be verified after construction is finished. Using this approach, previous experience can be used in combination with the reliability-based methods to stringently account for the predicted epistemic uncertainties in input parameters and to act as a basis for decision making in the initial design phase. In essence, the continuous observations performed during construction can be utilized to account for, and possibly reduce, the epistemic uncertainties incorporated in the design and construction of the support. Lastly, the analyzed structure can be continuously assessed, based on the results of the performed observations, by using the reliability-based methods to verify that the structure fulfills society's requirements on acceptable levels of safety. Thereby, environmental and economic optimization of the structure can be pursued, without compromising on the required levels of safety. This design approach is exemplified for limit states of type I in Appendix 1 and for type II in Bjureland et al. 2017.

This approach to tunnel design follows the conventional tunnel design process. To use experience from previously constructed tunnels and knowledge from the literature in the design process is no different from the approach that experienced engineers are used to apply in design of rock tunnel support, regardless of which safety assessment method that is used. When data are missing, experienced engineers commonly use their experience to assign values to the incorporated parameters that have not been quantified. The major difference lie in the quantification of the experience and the stringent use of it in the calculations. In deterministic calculation approaches, such as the total safety factor concept, uncertainties incorporated in the analyzed limit state are managed by the required SF. The magnitude of the uncertainty is then, however, in many cases unknown and, thus, the safety of the structure is essentially arbitrary.

When using the reliability-based methods as a safety assessment method, uncertainties are instead stringently accounted for in the analysis through the use of representative probability distributions for the input parameters. The reliability-based methods thereby highlight the effect of the uncertainties, and the limited knowledge inthe rock engineering industry, since they need to be quantified and defined. Thus, this approach encourages the engineer to gain better knowledge of the parameters incorporated in the analyzed limit states.

6.2.2 On the validity of the calculation models used in rock engineering design

When using reliability-based methods in combination with the limit states and calculation models used in today's design practice, as mentioned in Bjureland et al. (2017) and Bjureland et al. (2019), a question arises: are the calculation models calibrated to be used with reliability-based methods in combination with the $p_{\mathrm{f.traget}}$ specified in the Eurocodes (CEN 2002) and are their underlying assumptions valid?

Taking the analysis of shotcrete's flexural load bearing capacity against small loose blocks as an example, as described in Bjureland et al. (2019), Bjureland et al. (2020), and Appendix 1, two of the commonly used underlying assumptions are: (i) that the load-bearing capacity of the shotcrete is governed by the mean shotcrete thickness between four rockbolts and (ii) that the block, to which the shotcrete is exposed, can be treated as an evenly distributed load. That is, the stiffness of the rock mass that the block consist of is assumed to be equal to zero.

As illustrated in Appendix 1, both of these underlying assumptions can be questioned. By neglecting the spatial correlation of the shotcrete thickness by simply using the mean shotcrete thickness between four rockbolts to represent the thickness in the analysis of the shotcrete's flexural load-bearing capacity (Eq. 6), the load-bearing capacity of the shotcrete may be overestimated. On the contrary, if the stiffness of the loose block is accounted for in the analysis, the flexural load-bearing capacity of the shotcrete is in this case increased by a factor of three. This increase in load-bearing capacity is due to the relative stiffness of the block, compared to the shotcrete layer, and the block's capability to transfer the load from the center of the shotcrete between the four rockbolts towards the periphery of the block. As a result, the load is transferred closer to the rockbolts and the supporting edges of the shotcrete between them.

These findings highlight the limited knowledge that we in the rock engineering industry sometimes have in our calculation models and their applicability to the problem at hand. If reliability-based methods are to be used along with the $p_{\mathrm{f.traget}}$ required by the Eurocodes (CEN 2002), an increased knowledge regarding our calculation models is therefore necessary. This is by no means a question uniquely related to reliability-based methods. Of course, the question is highly relevant for the applicability of the Eurocode's version of the partial factor method and the total safety factor concept as well. The difference is that the Eurocode's version of the partial factor method and the total safety factor concept do not specify requirements on a specific $p_{\mathrm{f.traget}}$ that needs to be achieved.

6.2.3 System probability of failure and target probability of failure

The Eurocode specify that the provided values of $p_{\mathrm{f.traget}}$ must be obtained for individual components of a structure. Another question then arises: what is considered an individual component of a structure and what constitutes the structure? Again, taking shotcrete support of small loose blocks as an example, are the adhesive capacity, flexural capacity, punching shear capacity, and direct shear capacity all components of the structure? Or is the shotcrete support between four rockbolts a component of the complete tunnel structure?

As can be inferred from Appendix 1, shotcrete support of loose blocks between four rockbolts must be a component of the complete tunnel structure. Therefore, the limit states are not independent from each other and the correlation between them and their dependency on sufficient or insufficient adhesion must be accounted for, as shown in Bjureland et al. (2019). If not, the structural capacity of the analyzed support will be underestimated and as a result the calculated pf will be too high. The consequence would be an overly conservative design. The definition of what constitutes a structure and a component of a structure must be clarified in future revisions of the Eurocode.

6.2.4 Definition of structural failure

Another challenge of using reliability-based methods, as discussed in Bjureland et al. (2017), is to define what "failure" actually means in the context of rock tunnel design. Is failure the limit which causes a section of, or the whole, tunnel system to collapse if it is exceeded? Or is it maybe when yielding of one of the components included in the analyzed system occurs? As discussed in for example Johansson et al. (2016), the issue of defining failure is by no means only relevant when analyzing a tunnel using reliability-based methods. The same problem is present regardless of the method chosen for the analysis. Therefore, as suggested by for example Mašín (2015) and as essentially applied in this thesis, it might be more appropriate to define the limit state function as a limit of "unsatisfactory performance" instead of failure. However, when using this definition it must then be clarified what $p_{\mathrm{f.traget}}$ the engineer should use in a common practical design situation in order to fulfill society's demanded levels of safety.

7. Conclusions and suggestions for future work

7.1 Conclusions

The main purpose of this project has been to develop reliability-based design methods for environmental and economic optimization of rock tunnel support with a focus on shotcrete. To achieve this, the aim has been to: (1) assess the applicability of the partial factor method and reliability-based methods for design of shotcrete support, exclusively or in combination with the observational method, (2) quantify the magnitude and uncertainty of the shotcrete's input parameters, and (3) assess the influence from spatial variability on shotcrete's load-bearing capacity and judge the correctness of the assumption that the load- bearing capacity of the tunnel support is governed by the mean values of its input parameters; that is, it acts as an averaging system. Based on the research performed, the following can be concluded:

- The partial factor method is possible but not suitable to account for uncertainties in design of rock tunnel support for limit states with separable load and resistance (type I). For limit states with interaction between the load and the resistance (type II), the method is not applicable.

- The research also show that reliability-based methods are applicable to use in practical design of shotcrete to stringently account for the incorporated uncertainties in limit states of both type I and type II. A reliability-based design methodology for shotcrete support of loose blocks is presented in Appendix 1 and a reliability-based design methodology for shotcrete lining that fulfills the requirement of the observational method is presented in Bjureland et al. 2017.

- Based on the statistical quantification made in Paper B, suitable probability distribution for the input parameters governing the load-bearing capacity of shotcrete support was proposed. It can be concluded that for shotcrete thickness, the most suitable probability distribution is a lognormal distribution whereas for the adhesion, flexural tensile strength, residual flexural tensile strength and compressive strength it is the normal distribution.

- From the research presented in Bjureland et al. (2020), it can be concluded that using the spatial average of shotcrete thickness between four rockbolts to represent shotcrete thickness in design can result in an overestimation of the shotcrete lining's flexural load-bearing capacity. The spatial variability of shotcrete thickness therefore needs to be accounted for in the design work. In Bjureland et al. (2020),, it is further shown that if the minimum of (i) the spatial average thickness of a shotcrete slab of varying thickness, and (ii) the spatial average thickness of the slab along the circumference of the loose block, is used in design of shotcrete support the spatial variability of shotcrete thickness can be accounted for without complex and time-consuming numerical calculations.

- Further efforts need to be put into the definition of failure and how it relates to different limits of acceptable behavior. Alongside this, it should be clarified what the defined $p_{\mathrm{f.traget}}$ actually relates to.

- Information of parameters relevant to the design of underground excavations in rock, in terms of their representative distributions, is the basis for using reliability-based design. Therefore, additional gathering and quantification of data required in design of rock tunnel support from constructed rock tunnels along with further laboratory tests need to be performed. In particular, the spatial variability of the parameters needs to be quantified.

- A deeper review of the combination of using the limit states of today's practice in combination with reliability-based methods would be beneficial. In Bjureland et al. (2019), Bjureland et al. (2019), Bjureland et al. (2020), and Appendix 1, the possibility of using reliability-based methods for design of shotcrete support of loose blocks is illustrated and some of the basic assumptions are studied. However, the combination of reliability-based methods and limit states related to other failure modes need to be further addressed.

- Further research on the correctness of the calculation models used in the rock engineering industry is necessary. Taking the design of shotcrete support against loose blocks as an example, the uncertainty related to the relative stiffness between the block and the shotcrete layer is in Appendix 1 accounted for using a model factor. However, that model factor plays a crucial role in the reliability-based analysis of that limit state. Therefore the calculation model might not be appropriate to use in combination with the given $p_{\mathrm{f.traget}}$.

- Reliability-based methods have herein been successfully used in combination with analytical calculations. For complex design situations, numerical calculations might however be required. In future studies, reliability-based methods should therefore be combined with numerical calculations.

8. REFERENCES

Ang, A.H.-S. & Tang, W.H. 2007. Probability concepts in engineering: Emphasis on applications in civil & environmental engineering. Hoboken, NJ: John Wiley & Sons.

Baecher, G. & Christian, J. 2003. Reliability and statistics in geotechnical engineering. West Sussex: John Wiley & Sons Ltd.

Bagheri, M. 2011. Block stability analysis using deterministic and probabilistic methods. PhD thesis. Report nr TRITA-JOB PHD 1016. Stockholm: KTH Royal Insitute of Technology.

Bayes, T. 1763. An essay towards solving a problem in the doctrine of chances. By the late Rev. Mr. Bayes, F.R.S. communicated by Mr. Price, in a letter to John Canton, A.M.F.R.S. Philosophical transactions, 53, 370-418.

Bertsekas, D.P. & Tsitsiklis, J.N. 2002. Introduction to probability.

Belmont, MA: Athena Scientific Belmont.

Bjureland, W., Spross, J., Johansson, F., & Stille, H. 2015. Some aspects of reliability-based design for tunnels using observational method (EC7). In: Kluckner S, ed. EUROCK 2015. 1st ed. Salzburg; 2015:23-29.

Bjureland, W. 2017. On reliability-based design of rock tunnel support. Licentiate thesis. Report nr TRITA-JOB LIC 2033. Stockholm: KTH Royal Institute of Technology.

Bjureland, W., Spross, J., Johansson, F., Prästings, A. & Larsson, S. 2017. Challenges in applying fixed partial factors to rock engineering design. Geo-Risk 2017 GSP 283, 384-393.

Bjureland, W., Spross, J., Johansson, F., Prästings, A., & Larsson, S. 2017. Reliability aspects of rock tunnel design with the observational method. International Journal of Rock Mechanics and Mining Sciences 98, 102-110.

Bjureland, W., Johansson, F., Sjölander, A., Spross, J. & Larsson, S. 2019. Probability distributions of shotcrete parameters for reliability-based analyses of rock tunnel support. Tunnelling and Underground Space Technology 87, 15-26.

Bjureland, W., Johansson, F. & Spross, J. 2019. Spatial Variability of Shotcrete Thickness in Design of Rock Tunnel Support. 7th International Symposium on Geotechnical Safety and Risk, Taipei Taiwan, 11-13 December 2019.

Bjureland, W., Johansson, F., Spross, J. & Larsson, S. 2020. Influence of spatially varying thickness on load-bearing capacity of shotcrete. Tunnelling and Underground Space Technology 98, 103336.

Brady, B.H. & Brown, E.T. 2013. Rock mechanics: for underground mining. Springer Science & Business Media.

Brown, E.T., Bray, J.W., Ladanyi, B. & Hoek, E. 1983. Ground response curves for rock tunnels. Journal of Geotechnical Engineering, 109(1), 15-39.

Celestino, T.B., Aoki, N., Silva, R.M., Gomes, R.A.M.P., Bortolucci, A.A. & Ferreira, D.A. 2006. Evaluation of tunnel support structure reliability. In: L. In-Mo, Y. Chungsik & Y. Kwang-Ho (eds.), ITA- World Tunnel Congress, Seoul, 2006. Elsevier.

CEN 2002. EN 1990 - Basis of structural design. European comitteé for standardisation, Brussels.

CEN 2004. EN 1997-1:2004 Eurocode 7: Geotechnical design – Part 1: General rules. European committee for standardisation, Brussels.

Chang, Y. 1994. Tunnel support with shotcrete in weak rock – a rock mechanics study. PhD thesis. Stockholm: KTH Royal Institute of Technology.

Freudenthal, A. 1962. Safety, reliability and structural design.

Transactions of the American Society of Civil Engineers, 127(2), 304-318.

Freudenthal, A.M. 1947. The safety of structures. Transactions of the American Society of Civil Engineers, 112(1), 125-159.

Freudenthal, A.M., Garrelts, J.M. & Shinozuka, M. 1964. The analysis of structural safety. Columbia univerisity, institution for the study of fatigue and reliability, New york.

Goh, A.T.C. & Zhang, W. 2012. Reliability assessment of stability of underground rock caverns. International Journal of Rock Mechanics & Mining Sciences, 55, 157-163.

Goodman, R.E. & Shi, G.-h. 1985. Block theory and its application to rock engineering. Englewood Cliffs, NJ: Prentice-Hall.

Grimmelt, M.J. & Schuëller, G.I. 1982. Benchmark study on methods to determine collapse failure probabilities of redundant structures. Structural Safety, 1(2), 93-106.

Harr, M.E. 1987. Reliability-based design in civil engineering. New York, NY: McGraw-Hill.

Hasofer, A.M. & Lind, N.C. 1974. Exact and invariant second-moment code format. Journal of the Engineering Mechanics Division, 100(1), 111-121.

Hatzor, Y. 1992. Validation of Block Theory Using Field Case Histories.

PhD thesis. University of California, Berkeley.

Hibbett, Karlsson, Sorensen, 1998. ABAQUS/standard: User's manual.

Hibbitt, Karlsson & Sorensen.

Hochenbichler, M. & Rackwitz, R. 1981. Non-normal dependent vectors in structural safety. Journal of the Engineering Mechanics Division, 107(EM7), 1227-1238.

Hoek, E. & Brown, E.T. 1980. Underground excavations in rock.

Abingdon: Taylor & Francis.

Holmberg, M. & Stille, H. 2007. The application of the observational method for design of underground excavations. Stockholm: SveBeFo.

Holmberg, M. & Stille, H. 2009. The observational method and deformation measurements in tunnels. Stockholm: SveBeFo.

Homik, 2006. The R FAQ, http://cran.r-project.org, Accessed 2019-12- 16.

Jimenez-Rodriguez, R. & Sitar, N. 2007. Rock wedge stability analysis using system reliability methods. Rock Mechanics and Rock Engineering, 40(4), 419-427.

Jimenez-Rodriguez, R., Sitar, N. & Chacon, J. 2006. System reliability approach to rock slope stability. International Journal of Rock Mechanics and Mining Sciences, 43(6), 847-859.

Johansson, F., Bjureland, W. & Spross, J. 2016. Application of reliability- based design methods to underground excavation in rock. Stockholm: BeFo.

Klaube, M., 2018. Spatial Variability of shotcrete thickness. MSc thesis, TRITA–ABE–MBT 188. Stockholm: KTH Royal Institute of Technology.

Kohno, S. 1989. Reliability-based design of tunnel support systems. PhD thesis, 8924864. Urbana, Illinois: University of Illinois at Urbana-Champaign.

Krounis, A. 2016. Sliding stability re-assessment of concrete dams with bonded concrete-rock interfaces. PhD thesis. Report nr TRITA- JOB 2031. Stockholm: KTH Royal Institute of Technology.

Langford, J.C. & Diederichs, M.S. 2013. Reliability based approach to tunnel lining design using a modified point estimate method. International Journal of Rock Mechanics & Mining Sciences, 60, 263-276.

Laso, E., Lera, M.S.G. & Alarcón, E. 1995. A level II reliability approach to tunnel support design. Applied Mathematical Modelling, 19(6), 371-382.

Li, H.-Z. & Low, B.K. 2010. Reliability analysis of circular tunnel under hydrostatic stress field. Computers and Geotechnics, 37(1), 50- 58.

Lindfors, U., Swindell, R., Rosengren, L., Holmberg, M. & Sjöberg, J.

2015. Design of rock structures (Swedish: Projektering av bergkonstruktioner). Stockholm: Trafikverket.

Lloret-Cabot, M., Fenton, G.A. & Hicks, M.A. 2014. On the estimation of scale of fluctuation in geostatistics. Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 8(2), 129-140.

Low, B.K. 1997. Reliability analysis of rock wedges. Journal of Geotechnical and Geoenvironmental Engineering, 123(6), 498-

505.

Low, B.K. & Einstein, H.H. 2013. Reliability analysis of roof wedges and rockbolt forces in tunnels. Tunneling and underground space technology, 38, 1-10.

Lü, Q. & Low, B.K. 2011. Probabilistic analysis of underground rock excavations using response surface method and SORM. Computers and Geotechnics, 38(8), 1008-1021.

Lü, Q., Sun, H.-Y. & Low, B.K. 2011. Reliability analysis of ground- support interaction in circular tunnels using the response surface method. International Journal of Rock Mechanics & Mining Sciences, 48, 1329-1343.

Maidl, U., Köhler, M. & Schretter, K. 2011. Implementation of the observational method in mechanised tunnelling–contracts H3-4 and H8 in the Lower Inn Valley. Geomechanics and Tunnelling,

4(5), 405-413.

Mašín, D. 2015. The influence of experimental and sampling uncertainties on the probability of unsatisfactory performance in geotechnical applications. Géotechnique, 65(11), 897-910.

Massey Jr, F.J., 1951. The Kolmogorov-Smirnov test for goodness of fit.

Journal of the American Statistical Association. 46, 68-78.

Mauldon, M. 1990. Probability aspects of the removability and rotatability of tetrahedral blocks. International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts, 1990. Pergamon, 303-307.

Mauldon, M. 1992. Rotational failure modes in jointed rock: A generalization of block theory. PhD thesis. University of California, Berkeley.

Mauldon, M. 1993. Variation of joint orientation in block theory analysis. International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts, 1993. Elsevier, 1585-1590.

Mauldon, M. & Goodman, R. 1990. Rotational kinematics and equilibrium of blocks in a rock mass. International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts, 1990. Elsevier, 291-301.

Mauldon, M. & Goodman, R.E. 1996. Vector analysis of keyblock rotations. Journal of Geotechnical Engineering, 122(12), 976- 987.

Melchers, R.E. 1999. Structural reliability analysis. Chichester: John Wiley & Sons.

Nomikos, P.P. & Sofianos, A.I. 2010. An analytical probability distribution for the factor of safety in underground rock mechanics. International journal of rock mechanics & mining sciencs, 48(4), 597-605.

Palmstrom, A. & Stille, H. 2007. Ground behaviour and rock engineering tools for underground excavations. Tunnelling and Underground Space Technology, 22(4), 363-376.

Park, H.J., Um, J.-G., Woo, I. & Kim, J.W. 2012. The evaluation of the probability of rock wedge failure using the point estimate method. Environmental earth sciences, 65(1), 353-361.

Peck, R.B. 1969. Advantages and limitations of the observational method in applied soil mechanics. Géotechnique, 19(2), 171-187.

Quek, S. & Leung, C. 1995. Reliability based stability analysis of rock excavations. International journal of rock mechanics and mining sciences & geomechanics abstracts, 32(6), 617-620.

Schlater, M., 2001. Simulation of stationary and isotropic random fields. R-news 1, 18-20.

Shi, Y. & Stewart, M.G. 2015. Spatial reliability analysis of explosive blast load damage to reinforced concrete columns. Structural Safety, 53, 13-25.

Spross, J. 2014. A critical review of the observational method. Licentiate thesis. Report nr TRITA-JOB LIC 2024. Stockholm: KTH Royal Institute of Technology.

Spross, J. 2016. Toward a reliability framework for the observational method. PhD thesis. Report nr TRITA-JOB PHD 1023.

Stockholm: KTH Royal Institute of Technology.

Spross, J. & Johansson, F. 2017. When is the observational method in geotechnical engineering favourable? Structural safety, 66, 17- 26.

Spross, J., Johansson, F., Stille, H. & Larsson, S. 2014. Towards an improved observational method. In: L. R. Alejano, A. Perucho, C. Olalla & R. Jiménez (eds.), EUROCK 2014: Rock Engineering and Rock Mechanics: Structures in and Rock Masses, Vigo, Spain, 26-29 May 2014. London: Taylor & Francis group, 1435-1440.

Spross, J., Johansson, F., Uotinen, L.K.T. & Rafi, J. 2016. Using observational method to manage safety aspects of remedial grouting of concrete dam foundations. Geotechnical and Geological Engineering, 34(5), 1613-1630.

Spross, J. & Larsson, S. 2014. On the observational method for groundwater control in the Northern Link tunnel project, Stockholm, Sweden. Bulletin of Engineering Geology and the Environment, 73(2), 401-408.

Spross, J., Stille, H., Johansson, F. & Palmstrøm, A. 2018. On the need for a risk-based framework in Eurocode 7 to facilitate design of underground openings in rock. Rock Mechanics and Rock Engineering, 51(8), 2427-2431.

Statens Järnvägar. 1922. Statens Järnvägars geotekniska kommission 1914-22: Slutbetänkande avgivet till Kungl. Järnvägsstyrelsen den 31 maj 1922 [The Geotechnical Committee of the State Railways 1914-1922: Final report presented to the Royal Board of State Railways on 31 May 1922]. Stockholm: Statens Järnvägar.

Stille, H. 1986. Experiences of design of large large caverns in Sweden. In:

K. H. O. Saari (ed.), Proceedings of the International Symposium on Large Rock Caverns, Helsinki, 25-28 August 1986. Oxford, UK: Pergamon Press, 231-241.

Stille, H., Holmberg, M. & Nord, G. 1989. Support of weak rock with grouted bolts and shotcrete. International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts, 1(26), 99-113.

Stille, H., Holmberg, M., Olsson, L. & Andersson, J. 2005. Probability based design methods for underground excavations characterised by rock–structural interaction. Stockholm: SveBeFo.

Tonon, F. 1998. Generalization of Mauldon's and Goodman's vector analysis of keyblock rotations. Journal of geotechnical and geoenvironmental engineering, 124(10), 913-922.

Tonon, F. 2007. Analysis of single rock blocks for general failure modes under conservative and non-conservative forces. International Journal for Numerical and Analytical Methods in Geomechanics, 31(14), 1567-1608.

Vanmarcke, E. 1977. Probabilistic Modeling of Soil Profiles. Journal of the Geotechnical Engineering Division, 103(11), 1227-1246.

Vrouwendeler, A.C.W.M. 2002. Developments towards full probabilistic design codes. Structural Safety, 24(2), 417-432.

Welch, B.L., 1947. The generalization of student's' problem when several different population variances are involved. Biometrika, 34, 28- 35.

Zetterlund, M., Norberg, T., Ericsson, L.O. & Rosén, L. 2011. Framework for value of information analysis in rock mass characterization for grouting purposes. Journal of Construction Engineering and Management, 137(7), 486-497.

Zhang, W. & Goh, A.T.C. 2012. Reliability assessment on ultimate and serviceability limit states and determination of critical factor of safety for underground rock caverns. Tunnelling and Underground Space Technology, 32, 221-230.

Box 5501
SE-114 85 Stockholm
info@befoonline.org • www.befoonline.org
Visiting address: Storgatan 19, Stockholm
ISSN 1104-1773